**Binomial Distribution**

**Q1. 10 unbiased coins are tossed simultaneously ,find the probability that there will be i) exactly 5 heads ii) At least 8 heads iii) not more than 3 heads iv) At least one head. V) If this exercise is carried out 50 times, how many times we can get exactly 5 heads?**

Solution: The formula of binomial distribution is

P(r) = n C r p r q n-r

1. **Exactly 5 Heads**

n= 10, r =5, p =1/2, q =1/2

P(r=5) = 10 C 5 (1/2) 5 (1/2) 10-5

**=** 10 C 5 (1/2) 10

= 1/1024 \*10 C 5

= 1/1024 \* 10!/5!\*(10-5)!

3 2

1 10\*9\*8\*7\*6\*5!

= X

1024 5! x 5\*4\*3\*2\*1

**= 252/1024 = 0.246**

1. **At least 8 heads**

r>= 8, n= 10, p=1/2, q=1/2

The formula of binomial distribution is

P(r) = n C r p r q n-r

P(r>=8) = P(r=8) or P(r=9) or P(r=10)

P(r>=8) = P(r=8) + P(r=9) + P(r=10)

P(r>=8) = 10 C 8 (1/2) 8 (1/2) 10-8 + 10 C 9 (1/2) 9 (1/2) 10-9 + 10 C 10 (1/2) 10 (1/2) 10-10

**=** ( 1/2)10 10 C 8 +10 C 9 +10 C 10

= (45 +10+1)/ 1024 =56/1024 = **0.0546**

1. **Not more than 3 heads**  
   r<=3, n=10, p=1/2, q=1/2

The formula of binomial distribution is

P(r) = n C r p r q n-r

P(r<= 3) = P(r=0) or P(r=1) or P(r=2) or P(r=3)

P(r<= 3) = P(r=0) + P(r=1) + P(r=2) + P(r=3)

P(r>=3) = 10 C 0 (1/2) 0 (1/2) 10-0 + 10 C 1 (1/2) 1 (1/2) 10-1 +  10 C 2 (1/2) 2 (1/2) 10-2 +  10 C 3 (1/2) 3 (1/2) 10-3

**= ( 1/2)10 10 C 0 +10 C 1 +10 C 2 +10 C 3**

5 3

= (1+10+45+ 10\*9\*8\*7!/3\*2\*1\*7!) = (1+10+45+120)/1024

= 176/1024 = **0.1718**

**.0**

1. At least one head

r>=1, n=10, p= ½, q=1/2

P(r>=1) = 1- P(r=0) = 1- **10 C 0 (½) 0 (1/2) 10-0**

**= 1-1/1024 = 1024-1/1024 = 1023/1024 = 0.9999**

F(r) = N\*P(r=5) = 50\*0.246 = 12.3 times. 12 times -5 head

Q2. The mean and variance of a Binomial distribution are 3 and 2 respectively. Find the probability that the variant takes values i) exactly 2.ii) At most 2.

**Solution**: The mean of binomial distribution = np =3

And variance = npq =2

Variance /mean = npq/np = 2/3

q = 2/3, p+q =1, p+2/3=1, p = 1-2/3 = 3-2/3 =1/3.

np =3

n\*1/3 = 3, n = 3\*3 = 9

Q3. The incidence of a certain disease is such that on average, 20% of workers suffer from it. If 10 workers are selected at random find the probability that

1. Exactly 2 workers suffer from the disease
2. Not more than 2 workers suffer from the disease
3. At least 9 workers suffer from the disease

Solution: p =20% = 20/100 = 0.2, q = 1-0.2 = 0.8 , n= 10

1. r = 2
2. r <= 2
3. r >= 9

**Poisson Distribution**

Q1.  **If 5% of electric bulbs manufactured by a company are defective, Use Poisson Distribution to find the probability that in a box of 100 bulbs. i) None is defective ii) 3 bulbs are defective iii) More than 3 bulbs are defective. (Given= e -5 = 0.007).**

**Solution:**

The Poisson probability distribution function is

e –m m r

P(r) =

r!

where, m = np.

From this question, n =100, p =5% = 5/100 = 0.05

m= np = 100\*0.05 = 5

. i) None is defective ( r =0)

e –m m r

P(r) =

r!

e –5 (5) 0

P(r=0) =

0!

P(r = 0) = e –5

P(r = 0) = 0.007

ii) 3 bulbs are defective ( r =3)

e –m m r

P(r) =

r!

e –5 (5) 3

P(r=3) =

3!

0.007\* 125

P(r=3) =

6

P(r=3) = 0.1458

1. More than 3 ( r>3)

P(r>3) = 1- [P(r=0) +P(r=1) +P(r=2) +P(r=3)]

P(r>3) = 1- [P(r=0) +P(r=1) +P(r=2) +P(r=3)]

e –5 (5) 0 e –5 (5) 1 e –5 (5) 2 e –5 (5) 3

P(r>3) = 1- + + +

0! 1! 2! 3!

P ( r> 3) = 1- [ 0.007 + 0.007\*5+0.007\*25/2+0.1458]

P( r>3 ) = 1- 0.2753 = 0.7247

Q2. In a certain factory turning out razor blades, there is a small chance 1/500 for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate approximately, the number of packets containing

1. No defectives ii) Two defectives blades, in a consignment of 10,000 packets.

Solution: In the problem the number of blades in a packet = 10,

so, n= 10, p = 1/500, p =0.002, Hence, Mean = np = 10\* 0.002 = 0.02

The formula of Poisson distribution is :

e –m m r

P(r) =

r!

1. No blade is defective (r =0)

e –0.02 (0.02) 0

P(r=0) =

0!

P(r=0) = e – 0.02

**P(r =0) = 0.9802**

1. Two defectives blades

The formula of Poisson distribution is :

e –m m r

P(r) =

r!

e –0.02 (0.02) 2

P(r=2) =

2!

**P(r =2) = 0.9802\*0.0004/2 =** 0.00019604

So the number of defective packets in a consignment of 10,000 packets

= 0.00019604\*10000 = 1.96 **≅** 2 Packets

**Normal Distribution**

For all practical purpose we use the substitution, z = ( x-m)/ σ . Where z is called as standard Normal variate

Q1. The weekly wages of 1000 workers are normally distributed around a mean of Rs 70. and standard deviation of Rs 5. Estimate the number of workers whose weekly wages will be ;   
i) between Rs 70 and 72. ii) between Rs 69 and 72. iii) More than Rs. 75   
 iv) Less than Rs 63 v) Also estimate the lowest weekly wages of the 100 highest paid workers,

**Solution:** The mean value of the weekly wages is Rs 70 and standard deviation is Rs 5. To convert the variables into standard normal variate (z), will use the formula:

x- m

Z =

σ

Where x is the weekly wages, m is the mean value and σ is the standard deviation of the workers.

1. Estimate the number of workers whose weekly wages will be between

Rs 70 and Rs 72.

The value of z1 , when x1 = 70,

x1- m

z1 =

σ

70- 70

z1 = = 0/5 = 0

5

The value of z2 , when x2 = 72,

x2- m

z2 =

σ

72- 70

z2 = = 2/5 = 0.4

5

P(x1 ≤x ≤x2) = P(z1 ≤z ≤z2)

- ∞ 0 0.4 +∞

P(70 ≤x ≤72) = P(0 ≤z ≤0.4) = 0.1554.

The number of workers whose wages is between Rs 70 and Rs 72 is 0.1554\*1000= 155.4 ≅ 155 workers.

1. Estimate the number of workers whose weekly wages will be between

Rs 69 and Rs 72.

Solution : The value of z1 , when x1 = 69,

x1- m

z1 =

σ

69- 70

z1 = = -1/5 = -0.2

5

The value of z2 , when x2 = 72,

x2- m

z2 =

σ

72- 70

z2 = = 2/5 = 0.4

5

P(x1 ≤x ≤x2) = P(z1 ≤z ≤z2)

- ∞ -0.2 0 0.4 +∞

P(69 ≤x ≤72) = P(-0.2≤z ≤0.4) = P(-0.2≤z ≤ 0) + P(0≤z ≤ 0.4)

From the law of symmetry of normal distribution,

P(-0.2≤z ≤ 0) = P(0 ≤z ≤ 0.2)

**=** P(0 ≤z ≤ 0.2) + P(0≤z ≤ 0.4)

= 0.07926 + 0.15542 = 0.2346 ≅ 0.235

The number of workers whose wages is between Rs 69 and Rs 72 is 0.235\*1000= 235 workers.

iii) More than Rs. 75

Solution: The value of z1 , when x1 = 75,

x1- m

z1 =

σ

75- 70

z1 = = 5/5 = 1

5

P(x≥x1) = P(z≥z1)

P(x≥75) = P(z≥ 1) = P(1≤ z ≤ ∞)

- ∞ 0 1 +∞

**=** P (1 ≤z ≤ ∞) = P (0 ≤z ≤ ∞) - P (0 ≤z ≤ 1) = 0.5 – 0.3413 = 0.1587≅ 0.159

The number of workers whose wages more than 75 are 0.159\*1000= 159 workers.

iv) Less than Rs. 63

Solution: The value of z1 , when x1 = 63,

x1- m

z1 =

σ

63- 70

z1 = = -7/5 = -1.4

5

P(x≤x1) = P(z≤z1)

P(x≤63) = P(z≤ -1.4) , from the law of symmetry of normal distribution,

= P(z ≥1.4) = P(1.4 ≤ z ≤ ∞)

- ∞ 0 1.4 +∞

**=** P (1.4 ≤z ≤ ∞) = P (0 ≤z ≤ ∞) - P (0 ≤z ≤ 1.4) = 0.5 – 0.4192 = 0.0808≅ 0.081

So, the number of workers whose weekly wages is less than Rs 63 = 0.081\*1000=81 Workers

1. **Also estimate the lowest weekly wages of the 100 highest paid workers.**

**Solution:**

The proportion of 100 highest paid workers in total of 1000 workers = 100/1000=0.1

Hence, if x1 is the lowest weekly wages of the 100 highest paid workers, then we have to find, x1 such that

P (x≥x1) =0.1

x-m

Standard Normal Variate z =

SD

x1-70

Now, when x = x1, z1 =

5

Thus, we have to find z1 such that P (z≥z1) =0.1

* ∞ 0 z1 ∞

**⇒**P (z1≤z≤∞) =0.1

**⇒**P (0≤z≤∞) - P (0≤z≤z1) = 0.1

**⇒ 0.5**-P (0≤z≤z1) =0.1

**⇒** -P (0≤z≤z1) =0.1-0.5= -0.4

**⇒**P (0≤z≤z1) = 0.4

Reading the value of z1 corresponding to the probability (area under the normal curve) at 0.4 from the table in the reverse order we see that, z1= 1.28

x1-70

So, we have z1 =

5

x1-70

**⇒** 1.28 =

5

⇒ 1.28\*5 = x1-70

⇒ 6.4 = x1-70

⇒ 6.4+70 = x1

So, x1 = 76.4

The lowest weekly wages of 100 highest paid workers is Rs 76.40.